**The Aftermath of Accelerating Algebra:
Evidence from a District Policy Initiative**

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Abstract

In 2002/03, the Charlotte-Mecklenburg Schools initiated a broad program of accelerating entry into algebra coursework. The proportion of moderately-performing students taking 8th grade algebra increased from less than half to nearly 90%, then reverted to baseline levels, in the span of just six age cohorts. We use this policy-induced variation to infer the impact of accelerated entry into algebra on student performance in math courses as students progress through high school. Students affected by the acceleration initiative scored significantly lower on end-of-course tests in Algebra I, and were either no more likely or significantly less likely to pass standard follow-up courses, Geometry and Algebra II, on a college-preparatory timetable. We also find that the district assigned teachers with weaker qualifications to Algebra I classes in the first year of the acceleration, but this reduction in teacher quality accounts for only a small portion of the overall effect.

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1. Introduction

In 2008, the California State Board of Education voted to require all students to enroll in Algebra by 8th grade.[1] This policy initiative, yet to be actually implemented, represents the culmination of a decades-long movement toward offering algebra instruction before the traditional high school years.[2] Nationally, the proportion of eighth-grade students enrolled in algebra doubled between 1988 and 2007 (Perie, Moran and Lutkus, 2005; Walston and McCarroll 2010), reaching rates over 50% in three states and the District of Columbia.[3] The movement to offer algebra instruction before high school has been inspired in large part by correlational research documenting significant differences in later-life outcomes between those students who enroll in algebra by 8th grade and those who do not.

Correlation need not imply causation, and it is unclear whether accelerated algebra enrollment yields positive or negative effects (Loveless, 2008). This paper provides a quasi-experimental estimate of the causal impact of accelerating the introduction of algebra coursework. We analyze a policy initiative introduced in one of the nation's best-performing large school districts, Charlotte-Mecklenburg Schools (CMS), in 2002/03.[4] This initiative led students at many points in the initial math achievement distribution to take Algebra I earlier than they would have at baseline. After maintaining the acceleration policy for two years, the district reversed course, reverting almost entirely to its previous placement pattern. We use the across-

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1 This vote came at the urging of then-governor Arnold Schwarzenegger, who referred to algebra as "the key that unlocks the world of science, innovation, engineering, and technology. See "California to Require Algebra Taught in 8th Grade," *USA Today*, July 11, 2008; Eddy Ramirez, "8th-Grade Algebra Requirement in California Gets Sidelined," *U.S. News and World Report*, December 29, 2008.

2 In this paper, we use the term algebra to refer generically to a content area in mathematics, Algebra to refer to a course focusing on this content area, and Algebra I to refer to the course traditionally taken at the beginning of a college-preparatory math sequence in North Carolina public schools. We similarly distinguish between Geometry courses and the content area known as geometry.

3 In 2007, early algebra-taking rates exceeded 50% in California, Maryland, Utah, and the District of Columbia (Loveless, 2008).

4 Charlotte-Mecklenburg ranked first among the set of large American school districts with district-specific reports from the National Assessment of Educational Progress (NAEP) in terms of 4th grade mathematics scores for all test administrations between 2003 and 2009.

cohort variation in placement patterns created by these abrupt shifts in policy to infer the impact of acceleration, by comparing students with similar initial math achievement who were subjected to different placement policies solely on the basis of their age cohort.

We examine whether acceleration increased the likelihood that students would stay on track to pass three college-preparatory math courses – Algebra I, Geometry, and Algebra II – within six years of beginning seventh grade. Students who do so also meet the North Carolina State Board of Education's minimum standards for a college-preparatory course of study.[5] We use standardized end-of-course tests designed by the state to assess performance in each course, rather than the grade assigned by the course's instructor.[6]

Our results indicate that Charlotte-Mecklenburg's acceleration initiative worsened the Algebra I test scores of affected students and reduced their likelihood of progressing through a college-preparatory curriculum. Moderately-performing students who were accelerated into Algebra I in 8th grade scored one-third of a standard deviation worse on the state end-of-course exam, were 18 percentage points less likely to pass Geometry by the end of 11th grade, and were 11 percentage points less likely to pass Algebra II by the end of 12th grade, compared to otherwise similar students in birth cohorts that were not subjected to the policy. Lower-achieving students who were accelerated into taking the course in 9th grade also exhibited significant declines in all outcomes considered. By contrast, higher-performing students who were accelerated into Algebra I in 7th grade, despite receiving lower test scores on the Algebra I test, showed no ill effects on subsequent course completion.

[5] The State Board of Education permits a student to substitute a more advanced mathematics course – one using Algebra II as a prerequisite – for Geometry, or an alternative course sequence labeled Integrated Math I, II, and III in the state's official curriculum guide. In practice, the full Integrated Math sequence was not offered by any school in CMS during the period of study. Note also that admission to the 16-campus University of North Carolina system for most of the cohorts in our study required additional coursework beyond Algebra II. Thus completion of the three-course sequence was neither necessary nor sufficient for college admission. Nonetheless, failure to pass Algebra II effectively guaranteed that a student would not meet state standards for college-readiness.

[6] The state mandates that at least of the course grade in one of these courses be based on the end-of-course score. See GreatSchools, "Testing in North Carolina," http://www.greatschools.org/students/local-facts-resources/435-testing-in-NC.gs, 1/11/12.

The contrast between these results and prior correlational research reflects the severe selection bias plaguing those previous studies. It is undeniable that students who take algebra early tend to do better in subsequent math courses, but this correlation arises because it is usually the best students who are selected to take algebra early. Once this selection bias is eliminated, the remaining causal effect of accelerating the conventional first course of algebra into earlier grades, in the absence of other changes in the math curriculum, is for most students decidedly harmful. We caution that our results apply to the impact of varying the timing of the conventional first course in algebra, holding math instruction in the early grades constant. It is quite possible that more systematic intervention to transform the math curriculum at earlier ages to promote readiness for algebra by 8th grade could well prove beneficial.

2. Origins of the Algebra Acceleration Movement

As suggested by the brief history sketched above, accelerating algebra instruction into middle school has been widely espoused as a strategy for improving the mathematics achievement and college-readiness of American high school students. Nationwide, the proportion of 13-year-olds enrolled in algebra courses rose from 16% in 1988 to 29% in 2004 (Perie, Moran, and Lutkus, 2005). Among students in the nationally representative Early Childhood Longitudinal Survey Kindergarten cohort, just over one-third were enrolled in either algebra or a more advanced math course in 2006/07, when most of the cohort was in 8th grade (Walston and McCarroll, 2010). As noted above, this national average obscures very high rates of 8th grade algebra-taking in some jurisdictions.

This movement has been supported in part by assigning a causal interpretation to correlational research. Eighth grade students enrolled in algebra outscore their counterparts on 8th grade standardized math tests (Walston and McCarroll, 2010). By the time they reach 12th grade, early algebra-takers have completed more years of advanced math and attain higher scores on 12th grade math assessments (Smith, 1996). Additional research has documented higher achievement outcomes among students who enroll in algebra at any point in their secondary school career (Dossey et al., 1988; Gamoran and Hannigan, 2000). Ma (2005a; 2005b) reports that the improvement in math skills associated with enrollment in 8th grade algebra is strongest for the lowest-achieving students – particularly those below the 65th percentile of the 7th grade math distribution. To date, no study has attempted to address concerns regarding selection into accelerated algebra on the basis of unobserved characteristics.[7]

Indeed, doubts about the reliability of previous studies have provoked a backlash against accelerating algebra into middle school. Opponents of accelerated algebra argue that too many students enter the course unprepared for advanced work and may in fact fall behind their peers enrolled in less rigorous coursework. Loveless (2008) documents the poor math performance of some students enrolled in the course by 8th grade, and he notes the inattention to the problem of selection in prior work justifying the push to offer algebra in middle school. The Loveless report itself, however, provides no evidence on the causal question of whether early placement in algebra promotes or retards mathematics achievement. The poorly-performing students he cites may have performed just as poorly in a more traditional 8th grade math course. An empirical assessment of the effects of accelerating the first algebra course requires observation of a counterfactual scenario: otherwise identical students who take algebra on a traditional schedule.

3. Conceptual Framework

3.1 Algebra timing, mathematics skills, and labor productivity

[7] Ma (2005b), for example, reports that only 4% of students below the 65th percentile of the 7th grade math distribution are placed in algebra by 8th grade.

From an economic perspective, algebra skills can be valued for two basic reasons. First, algebra skills may contribute directly to labor productivity.[8] Second, algebra skills might serve as inputs into the production of higher-order mathematical knowledge, which in turn may have an independent effect on productivity. The notion that algebra is a "gateway" derives from this second interpretation. These two effects on productivity can be summarized in this expression:

(1) $y = y(a(t_a), h(a, t_h))$,

where y is a measure of productivity, a is a measure of algebra skill, h is a measure of higher-order mathematical skill, and t_a and t_h measure the amount of time devoted to the study of algebra and higher-order topics, respectively. All three functions in equation (1) are presumed to be nondecreasing in their arguments. If students are expected to complete their human capital investment by a specific age, the case for accelerating entry into algebra is clear: initiating algebra earlier allows more time for instruction in both algebra and higher-order topics, thereby unambiguously increasing productivity.

Things get more complicated when we introduce the possibility that both algebra and higher-order math skills rely on the degree to which students have mastered lower-order topics in mathematics. Consider the formulation:

(2) $y = y(l(t_l), a(l, t_a), h(a, l, t_h))$

where l and t_l represent lower-order mathematical skill and the time devoted to learning these skills, respectively. While we did not introduce an explicit time constraint in our initial formulation, it makes sense here to assume a fixed amount of time available between school entry and the end of human capital investment. In this formulation, the opportunity cost of accelerating introduction to algebra is clear. Indeed, the question of algebra timing reduces to a matter of how much time to allocate to lower-order subjects. The belief that students enter

[8] Beyond improving labor productivity and earnings, math skills may also increase utility by promoting better consumption decisions by boundedly-rational agents (Benjamin, Brown, and Shapiro, 2006).

algebra too late is equivalent to an argument that too much time is devoted to lower-order subject matter.

Equation (2) implies that the optimal allocation of time across mathematical topics depends on a number of relationships: the relative importance of lower-order skills in the production of higher-order skills, the marginal impact of time on skill acquisition, and the relative importance of various types of mathematical skill on productivity. A proposal to reallocate time away from lower-order skills makes the most sense if lower-order skills are relatively unimportant in the production of algebra and higher-order skills, and if lower-order skills are similarly unimportant determinants of productivity.

3.2 The opportunity cost of acceleration

What kinds of topics are short-changed when algebra is accelerated? To get an idea, Table 1 describes the key competencies that North Carolinas' standard course of study establishes for several pre-algebra courses, ranging from 7th Grade Math to Introductory Math, the course prescribed for students who do not take Algebra I upon entry into high school.[9]

The similarity in course objectives across 7th and 8th grade math, and the high school introductory math course, suggests the possibility of diminishing returns in lower-order mathematics instruction. The objectives of 8th grade math and Introductory Math are nearly identical, suggesting that the high school course largely repeats subject matter for students who did not master it the first time around. Furthermore, the distinctions between 7th and 8th grade math objectives are minor; eighth graders, for example, are expected to perform computations

[9] These competencies form the basis for standardized End-of-Grade tests in mathematics conducted since the early 1990s.

with irrational numbers whereas in seventh grade computation with rational numbers is sufficient.[10]

Although a perusal of these stated objectives suggests that pre-algebra courses are incremental if not redundant, it is possible that many students need repeated exposure to this subject matter. It is interesting to note, furthermore, that each of the middle-grades math courses includes significant attention to geometry. Computation of volume and surface area is a key component of the 7th grade curriculum, and the Pythagorean theorem is mentioned specifically in the 8th grade curriculum. Both topics appear in the high school Introductory Math course, and both relate directly to subjects covered in the state's official Geometry curriculum, which focuses in part on right triangles, problems involving surface area and volume, and elementary proof-writing.

Algebra I acceleration is not the only curricular reform designed to improve mathematics achievement. California's Math A and New York's Stretch Regents curriculum are examples of reforms that target the quality of pre-algebra instruction rather than the timing of algebra coursetaking (White 1995; White et al. 1996; Gamoran et al. 1997).[11] Although evidence on the effectiveness of these programs is inconclusive (White et al 1996; Gamoran et al, 1997), these alternatives may offer promising avenues to improve achievement in the event that accelerating algebra is judged not to be worth the cost of forgone pre-algebra instruction.

The larger question of which math subjects have the strongest effects on productivity is beyond the scope of our empirical analysis, although it certainly bears heavily on the question of optimal time allocation. In one study pertinent to this issue, Rose and Betts (2004) analyze transcript data from the High School and Beyond dataset, based on straightforward methods to

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[10] A rational number is one that can be expressed as the ratio of two integers.
[11] Math A is a high school curriculum used in certain districts used to transition lower achieving students to a college-preparatory algebra and geometry curriculum. The Stretch Regents program permits students to take New York State's rigorous Regents curriculum at a slower pace. See Gamoran (1997) for further description.

address concerns about self-selection into higher-order courses. This study suggests that the labor market return to higher-order coursework is greater than the return to coursework at the level of introductory algebra or geometry.

4. Data and Methodology

4.1 Setting

Our analysis makes use of data on students enrolled in the Charlotte-Mecklenburg Schools (CMS), provided by the North Carolina Education Research Data Center. During the period of our analysis, CMS was the largest school district in North Carolina, and one of the 25 largest in the United States, serving over 100,000 students. The district is racially diverse; in 2002/03, the first year of implementation for the Algebra acceleration program we study, 44% of all students were black, 8% were Hispanic, and 4% Asian. About 40% of the district's students participated in the federal free and reduced price lunch program, slightly above the state average.

Charlotte-Mecklenburg has a strong reputation for mathematics performance. The district's fourth grade students ranked first among 18 major school districts in the 2009 National Assessment of Educational Progress (NAEP) math assessment. It was the only district in this group with 4th grade math scores significantly higher than the national average. To put this high performance in context, however, it should be noted that, because it covers both suburban and urban neighborhoods, CMS has a larger share of middle class students than do most large school districts.[12]

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[12] Ranked by performance of students eligible for federal school lunch subsidies, CMS placed 4th among the 18 districts. Nonetheless, CMS presents a case of algebra acceleration in a large urban district with relatively strong math performance.

Beginning around 2002/03, CMS adopted an unusually aggressive policy to accelerate placement of middle and high school students in Algebra I.[13] The district not only broke from its past patterns of course-taking but also diverged dramatically from policies followed by most other districts in North Carolina. By all appearances, there were two precipitating factors that accounted for Charlotte-Mecklenburg's aggressive approach. First, the state of North Carolina had increased from three to four the number of math courses required for admission to the University of North Carolina system. Second, the district's then superintendent strongly believed as a matter of pedagogy that algebra should be offered to many, if not most, students in middle school, rather than waiting until they are in high school. Later described as "a bear on getting middle school kids in eighth grade to learn Algebra I," this superintendent announced at the beginning of the 2001/02 year that his goal would be to increase to 60% the portion of students in the district who were proficient in Algebra I by the end of eighth grade, as indicated by scoring at level 3 or above on the state's end-of-course test.[14]

Several other policy changes transpired in CMS during the period of our study. The districted ceased busing students to desegregate schools in 2002, and implemented a public school choice plan, incorporating a lottery system for oversubscribed schools the same year (Hastings, Kane, and Staiger 2005, 2006a, 2006b; Hastings et al. 2007; Deming et al. 2011; Vigdor 2011). These changes may have led to systematic declines in instructional quality for

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[13] Although we have found no written statement of Charlotte-Mecklenburg's policy, its existence and influence have been substantiated by contemporaneous reporting and the recollection of administrators who worked in the system at the time of implementation.

[14] *Educate!*, September 16, 2001, p. 5. As evidence of the superintendent's focus on increasing the number of middle school students taking algebra, one informant described how he ordered middle school principals to overhaul schedules after the school year had commenced in order to increase the number of middle school students in algebra classes. In an interview after he stepped down as CMS superintendent, Eric Smith stated, "The middle school math piece was the gatekeeper and it is the gatekeeper. It's the definition of what the rest of the child's life is going to look like academically, not just through high school but into college and beyond. If they make it into algebra one, the likelihood of getting into the AP class and being successful on the SAT and having a vision of going on to college is dramatically enhanced. And so our pressure to make sure that kids were given that kind of access to upper level math in middle school was a critical component of our overall district strategy." [14] http://www.pbs.org/makingschoolswork/dwr/nc/smith.html, 4/5/11.

African-American and other disadvantaged students (Jackson 2009). We detail below several strategies for addressing potential confounding effects. Most importantly, we obtain very similar results when analyzing a similarly-timed initiative in North Carolina's third-largest school district.

Figure 1 summarizes information on algebra-taking patterns in CMS for six age cohorts included in this study. It is based on a longitudinal sample of students described in more detail below.[15] For each student, we record the year in which he or she first takes North Carolina's end-of-course test in Algebra I.[16]

The initial cohort enrolled in 7th grade for the first time in 1999/2000, three years prior to the algebra acceleration initiative. At this time, rates of algebra-taking by 8th grade were high relative to the national average for high-performing students, but low for low-performing students. Ninety-seven percent of CMS students in the top quintile of the statewide 6th grade math score distribution were enrolled in Algebra I by 8th grade, compared to 75% of top quintile 8th graders nationwide, as recorded in the 2009 NAEP assessment (Walston and McCarroll 2010). By contrast, only 2% of CMS students in the lowest 6th grade math quintile had enrolled in Algebra I by 8th grade, compared to 13% in the national NAEP data.

The cohort entering 8th grade in 2001/02, just two years later, experienced a very different pattern. For this later cohort, the rate of early algebra-taking shifted dramatically at lower points in the distribution. For students around the median, the likelihood of taking algebra by 8th grade increased from 47% to 86%. For students in the second-lowest quintile, the rate

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[15] Note that all analyses reported in this paper "undo" effects of grade retention by comparing students only to those in their entering cohort. To be precise, therefore, our analyses study not the impact of taking Algebra I by 8th grade, but the effect of taking the course within two years of beginning seventh grade.

[16] More precisely, we present the year in which a student first appears as a data point in the Algebra I EOC test file. A small number of students appear in the dataset but do not have a valid test score. These students are excluded from analyses using test scores as a dependent variable below, but are included in analyses of subsequent course-taking.

increased from 14% to 65%. Even in the lowest quintile of the 6th grade math distribution, the rate of Algebra I taking rose to 18%.[17]

Just two years after the push to accelerate algebra started, however, the district reversed course. By the time the cohort that entered 7th grade in 2004/05 had reached middle school, assignment patterns were nearly back to those for the 1999/2000 cohort, except in the top two quintiles, where a modest amount of acceleration remained in place. This rapid reversal of the acceleration policy provides us with the first means of distinguishing acceleration effects from the effects of resegregation and school choice.

Figure 2 shows that the acceleration policy involved more than pushing students into 8th grade algebra. For certain students, the likelihood of taking Algebra I by 7th grade also increased substantially over time. In the 1999/2000 cohort, just under half of top quintile students, 12% of second quintile students, and 2% of middle quintile students took Algebra I as 7th graders. In the top quintile, the rate of 7th grade Algebra I enrollment rose monotonically, reaching 75% by 2005. In the second quintile, the 7th grade Algebra I-taking rate rose to nearly 40% in 2004 before retreating somewhat.

For students in the lowest two quintiles of 6th grade math test scores, the acceleration policy had its biggest effect in an increased propensity to take Algebra I by 9th grade. Figure 3 shows a peak among students entering 7th grade in 2000/01, who would have entered 9th grade in 2002/03 under normal rates of academic progress. Over 70% of lowest-quintile students in this cohort had taken Algebra I by 9th grade. But by the time the 2004/05 cohort came through, just

[17] Our data are derived from end-of-course test records, which may not accurately measure the number of students assigned to take Algebra I in a given year. Students may withdraw from the course in advance of test administration, for example. There is some evidence that the rate of withdrawal rose in 2002/03 along with the rate of course-taking. In that year, an administrative count of course enrollment in Algebra I for CMS enumerates over 900 students for whom we have no test score record. In most other years, the discrepancy between the two sources of enrollment data is small. We discuss potential implications of this pattern below.

over a third of students in the bottom quintile were getting this treatment. . Similar fluctuations occurred in the fourth and middle quintiles.

4.2 Data and Sample Selection

Our data are derived from North Carolina Education Research Data Center longitudinal records on students who entered 7th grade in the Charlotte-Mecklenburg district between 1999/2000 and 2004/05 and were observed in Algebra I EOC test score files for that district.[18] In most cases, we also restricted the sample to students with valid scores on the state's standardized 6th grade mathematics assessment.[19] We tracked progress through college-preparatory math courses using the state's end-of-course (EOC) examinations in Algebra I, Geometry, and Algebra II. Our ultimate sample consists of 36,790 students across six cohorts.[20]

4.3 Identification Strategy

Our estimation strategy takes advantage of the significant policy changes that took place in Charlotte-Mecklenburg over just a few years. We exploit these changes to estimate local average treatment effects for taking Algebra I by the time students reach a certain grade. We

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[18] This restriction allows us to focus exclusively on the question of whether accelerating students into algebra yields benefits, rather than the broader question of whether algebra instruction itself is beneficial. Results obtained with a broader sample of students are qualitatively similar to those reported here.

[19] We made exceptions to this restriction for the first cohort, wherein problems involving NCERDC identification codes greatly reduced matching rates from seventh backwards to sixth grade. For students with missing 6th grade information in this cohort, we assign them to an initial test score decile on the basis of their 7th grade assessment. We have estimated each of our regression specifications excluding these students; results are not significantly affected by their exclusion.

[20] Some of the students included in our sample may exit the dataset because they leave the CMS system, to attend a different public district, a private or charter school. If such students complete Geometry or Algebra II coursework, we will incorrectly code them in our analysis. Due to differences in student ID coding between CMS and other North Carolina districts, we are unable to satisfactorily track students who transfer to a different district or to a charter school. Moreover, given data limitations it is impossible for us to distinguish a student who attrits from one who persists without taking EOC exams. This poses a problem for our analysis only to the extent to which transfer behavior correlates with algebra acceleration, conditional on decile and cohort effects. If parents respond to the decline in mathematics performance associated with algebra acceleration by switching to a different school district, we may in fact overstate our results. Note that we are similarly unable to identify students who drop out of school; since students cannot pass EOC exams after dropping out, however, they are not miscoded.

begin by examining the effect of taking the course by 8th grade, and we later look at the effects of accelerating Algebra I into 7th grade or 9th for students at different points in the initial achievement distribution. The estimated treatment effects are "local" to that set of students subjected to differing treatment status across cohorts within our six-cohort sample. For example, our estimate of the effect of taking Algebra I by 8th grade applies primarily to students in the middle of the initial test score distribution; students at the top of the distribution virtually always take Algebra I by 8th grade, while those at the bottom rarely do.[21]

Our basic estimation strategy is a version of differences-in-differences: we compare the outcomes of students stratified by initial ability level, as measured by 6th grade math scores, across cohorts. In order to implement this strategy in a manner that produces local average treatment effects, we use instrumental variable estimators. The outcome equation is of the form:

(4) $y_{idc} = \alpha_c + \alpha_d + \beta T_{idc} + \varepsilon_{idc}$

where y_{idc} is the outcome of interest for student i belonging to initial achievement decile d in cohort c, α_c and α_d are cohort and decile fixed effects, T_{idc} is an indicator for whether the student received the treatment – in this case, taking Algebra I by a certain point in their career – and ε_{idc} is an independent and identically distributed error term. Cohort fixed effects account for policy changes or other contemporaneous effects that apply to all students in a cohort, while decile fixed effects account for broad differences in outcome trajectories for students with differing initial ability. The use of decile effects rather than a linear control for test score allows us to account for potentially nonlinear effects of initial ability on later outcomes.

[21] Our results may have some bearing on the most prominent algebra policy debate, regarding California's initiative to require 8th grade algebra. Note, however, that this initiative is most relevant for the bottom 40% of the math ability distribution in that state, since the rate of 8th grade algebra-taking is already close to 60%. Our estimate of the effect of 8th grade algebra acceleration pertains more directly to students towards the middle of the ability distribution. Our results on accelerating low-performing students into 9th grade algebra may provide some additional insight as to the effects of acceleration in that subset of the population.

Prior work in this literature has often estimated single equations along the lines of (4), arguing that controls for prior achievement adequately correct for unobserved determinants of the outcome that also correlate with the treatment indicator, implying that β is an unbiased estimate of the true treatment effect. To assess this argument, we present OLS estimates of equation (4) for comparison with our preferred IV results below.

In our preferred specifications, we address the endogeneity of assignment to an accelerated algebra class by estimating the first stage equation:

$$(5)\quad T_{idc} = \gamma_c + \gamma_d + \sum_{c=1}^{6}\sum_{d=1}^{10}\delta_{dc} + \eta_{idc}$$

where γ_c and γ_d are cohort and decile fixed effects, the δ_{dc} terms are cohort-by-decile fixed effects, and η_{idc} is a second error term. Predicted values of equation (5) are then used in place of actual treatment status in equation (4). Effectively, the estimation strategy associates across-cohort-and-decile variation in the propensity to take Algebra I by a certain grade level with across-cohort-and-decile variation in the outcome of interest. We attribute a positive (negative) effect to acceleration if students subjected to a higher probability of earlier algebra than others in the same initial ability decile in another cohort exhibit superior (inferior) performance in Algebra I and subsequent math courses. Because the identifying variation in algebra timing is at the cohort-by-decile level, we cluster standard errors at that level.

In principle, we would like to estimate the impact of early progression to Algebra I on performance in that course and subsequent math topics. This goal is complicated by the fact that many students who enroll in Algebra I do not complete subsequent math coursework. Thus, any effort to estimate the impact of Algebra I timing on performance in Geometry or Algebra II must contend with a sample selection problem: we can observe performance in subsequent math courses only for those who actually take and complete those subsequent courses. If acceleration